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Erratum

Erratum to "On the generation of P-stable exponentially fitted Runge–Kutta–Nyström methods by exponentially fitted Runge–Kutta methods"

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In this errata we fill a gap in the results of [2]. In that paper we have proved that a collocation-based exponentially fitted Runge–Kutta (EFRK) method with symmetric placed nodes generates a P-stable exponentially fitted Runge–Kutta–Nyström (EFRKN) method when $K \geqslant 1$. Here, we go one step further: we have proved that this result is also true in the case that $K = 0$.

EFRK methods characterized by $K = 0$ do not satisfy the usual RK assumptions

$$Ae = c \quad \text{and} \quad b^{\mathrm{T}} e = 1. \tag{0.1}$$

Without the restrictions (0.1), the generated RKN method takes the form

$$\begin{aligned} Y_i &= y_n + h y'_n \sum_{j=1}^{s} a_{ij} + h^2 \sum_{j=1}^{s} (a_N)_{ij} f(x_n + (c_N)_j h, Y_j), \quad i = 1, \ldots, s, \\ y_{n+1} &= y_n + h y'_n \sum_{i=1}^{s} (b_N)_i + h^2 \sum_{i=1}^{s} (\bar{b}_N)_i f(x_n + (c_N)_i h, Y_i), \\ y'_{n+1} &= y'_n + h \sum_{i=1}^{s} (b_N)_i f(x_n + (c_N)_i h, Y_i), \end{aligned} \tag{0.2}$$

where

$$c_N = c, \quad b_N = b, \quad A_N = A^2, \quad \bar{b}_N^{\mathrm{T}} = b^{\mathrm{T}} A. \tag{0.3}$$

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The *stability matrix* [1,3] of algorithm (0.2) is given by

$$M(H^2) = \begin{pmatrix} 1 - H^2 \bar{b}_N^{\mathrm{T}} N^{-1} \mathrm{e} & b_N^{\mathrm{T}} \mathrm{e} - H^2 \bar{b}_N^{\mathrm{T}} N^{-1} A\mathrm{e} \\ -H^2 b_N^{\mathrm{T}} N^{-1} \mathrm{e} & 1 - H^2 b_N^{\mathrm{T}} N^{-1} A\mathrm{e} \end{pmatrix}, \tag{0.4}$$

where $N = I_s + H^2 A_N$. The next theorem reveals the relation between the stability matrix and the stability function of a RKN method generated by a RK method. The relationship was previously found by van der Houwen et al. [3] and De Meyer et al. [1] for RK methods satisfying (0.1). The theorem remains valid also when the assumptions (0.1) are not satisfied. The proof is a slight adaptation of the proof of Theorem 13 of [1].

Theorem 1. *If a RKN method is generated by a RK method* (c, A, b), *then*

$$M(H^2) = R(Z) \quad \textit{with } Z = \begin{pmatrix} 0 & 1 \\ -H^2 & 0 \end{pmatrix},$$

where R *is the stability function of the RK method.*

Proceeding as in [2] we conclude with:

Theorem 2. *An EFRKN method generated by an EFRK method of collocation type with symmetric placed nodes is P-stable.*

In contrast with [2], Theorem 2 allows to consider two-stage methods. Let us note that at the end of paper [2], we have made the false conclusion that EFRK methods with $K = 0$ generate dissipative EFRKN methods and therefore they are not P-stable. Our mistake is due to the use of the stability matrix of conventional RKN methods. Finally, we have found a small error in the proof of Theorem 3 from [2]. On page 316, Step 2: “v” should be “v_n”.

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